VA Uniforme

$$f_X(x)=\begin{cases}\frac{1}{b-a} & a\le x\le b\\ 0 & else\end{cases}$$

$$F_X(x)=\begin{cases}0 & x<a\\ \frac{x-a}{b-a} & a\le x\le b\\ 1 & x>b\end{cases}$$

VA Gaussiana

$$f_X(x)=\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(x-\eta)^2}{2\sigma^2}}$$

$$F_X(x)=1-Q\left(\frac{x-\eta}{\sigma}\right)$$

$$f_{XY}(x,y)=\frac{e^{-\frac{1}{2(1-\rho^2)}\left[\frac{(x-\eta_X)^2}{\sigma_X^2}-2\rho\frac{(x-\eta_X)(y-\eta_Y)}{\sigma_X\sigma_Y}+\frac{(y-\eta_Y)^2}{\sigma_Y^2}\right]}}{2\pi\sigma_X\sigma_Y\sqrt{1-\rho^2}}$$

$$\Phi_X(s)=e^{s\eta+\frac{(\sigma s)^2}{2}}$$

VA Esponenziale

$$f_X(x)=\begin{cases}\lambda e^{-\lambda x} & x\ge 0\\ 0 & x<0\end{cases}$$

$$F_X(x)=(1-e^{-\lambda x})U(x)$$

$$E[X]=\frac{1}{\lambda}\qquad Var[X]=\frac{1}{\lambda^2}$$

$$\Phi_X(s)=\frac{\lambda}{\lambda-s}$$

VA Poisson

$$p_i=e^{-\lambda}\frac{\lambda^i}{i!}\qquad i=0,1,2,\ldots$$

$$E[X]=\lambda,\qquad Var[X]=\lambda$$

$$\Phi_X(s)=e^{\lambda(e^s-1)}$$

VA Binomiale

Numero di successi in *n* prove

$$p_i=\binom{n}{i}p^i(1-p)^{n-i}\quad i=0,1,2,\ldots n$$

$$E[X]=np\qquad Var[X]=np(1-p)$$

$$\Phi_X(s)=[(pe^s)+(1-p)]^n$$

VA Binomiale negativa

Numero di prove fino al *r-mo* successo

$$p_i=\binom{i-1}{r-1}p^r(1-p)^{i-r}\quad i=r,r+1,r+2\ldots$$

VA Pascal

Numero di insuccessi fino al *r-mo* successo

$$p_i=\binom{i+r-1}{i}p^r(1-p)^i\qquad i=0,1,2,\ldots$$

Formule di Bayes

$$P(A\mid E)=\frac{P(E\mid A)\cdot P(A)}{P(E)}$$

$$P(M\mid X=x)=\frac{f_X(x\mid M)}{f_X(x)}\cdot P(M)$$

PT: $f_X(x)=\sum_{i=1}^{+\infty}f_X(x\mid A_i)\cdot P(A_i)$

$$E[X]=\sum_{i=1}^{+\infty}E[X\mid A_i]\cdot P(A_i)$$

$$E[g(X)]=\sum_{i=1}^{+\infty}E[g(X)\mid A_i]\cdot P(A_i)$$

TA: $E[g(X)]=\int_{-\infty}^{+\infty}g(x)f_X(x)dx$

$$E[X]=\int_0^{+\infty}[1-F_X(x)]dx+\int_{-\infty}^{0}F_X(x)dx$$

$$E[g(X,Y)]=\begin{cases}\int_{-\infty}^{+\infty}g(x,y)f_{XY}(x,y)dxdy & cont\\ \sum_i\sum_k g(x_i,y_k)p_{XY}(x_i,y_k) & disc\end{cases}$$

$$E[g(X,Y)]=\int_{-\infty}^{+\infty}f_Y(y)\left[\sum_i g(x_i,y)p_X(x_i\mid Y=y)\right]dy$$

$$=\sum_i\left[\int_{-\infty}^{+\infty}g(x_i,y)f_Y(y\mid X=x_i)dy\right]p_X(x_i)$$

X discreta e Y continua

Linearità della media: $E[aX+bY]=aE[X]+bE[Y]$

TMC: $E[X]=E_Y[E_X[X\mid Y]]$

Trasformazioni notevoli

$Y=F_X(X)$, $\forall X, Y\sim Unif(0,1)$

$$Z=X+Y\qquad f_Z(z)=\int_{-\infty}^{+\infty}f_{XY}(z-u,u)du$$

$f_Z(z)=f_X\otimes f_Y=\int_{-\infty}^{+\infty}f_X(z-u)f_Y(u)du$ (X,Y) indip.

$$Z=\frac{X}{Y}\qquad f_Z(z)=\int_{-\infty}^{+\infty}|u|f_{XY}(zu,u)du$$

Coord polari

$$f_{r\theta}(r,\theta)=\begin{cases}rf_{XY}(r\cos\theta,r\sin\theta) & r>0\wedge\theta\in[-\pi,\pi]\\ 0 & else\end{cases}$$

$$Y=a\sin(X+\theta),X\sim Unif(-\pi,\pi)\to f_Y(y)=\frac{1}{\pi\sqrt{a^2-y^2}}$$

<u>Trasformazioni lineari di vettori Gaussiani danno ancora vettori Gaussiani</u>

Teorema Fondamentale

$$Y=g(X)\qquad f_Y(y)=\frac{f_X(x_1(y))}{|g'(x_1(y))|}+\ldots+\frac{f_X(x_n(y))}{|g'(x_n(y))|}$$

$$\begin{cases}Z=g(X,Y)\\ W=h(X,Y)\end{cases}\quad f_{ZW}(z,w)=\frac{f_{XY}(x_1,y_1)}{|J(x_1,y_1)|}+\ldots+\frac{f_{XY}(x_n,y_n)}{|J(x_n,y_n)|}$$

con (x_i,y_i) soluzioni del sistema e J lo Jacobiano (gradiente di z e w, risp a (x,y))

Correlazione di X e Y $E[XY]$, Incorr. se $E[XY]=E[X]E[Y]$

$Cov[X,X] = Var[X] \qquad Cov[X,a] = 0 \quad \forall a \in R$

$Var[X] = E[X^2] - \eta_X{}^2 \qquad Cov[X,Y] = E[XY] - \eta_X \eta_Y$

$$\rho_{XY} = \frac{Cov[X,Y]}{\sqrt{Var[X]Var[Y]}} = \frac{C_{XY}}{\sigma_X \sigma_Y}$$

Se X,Y sono linearmente dipendenti, allora $|\rho_{XY}| = 1$

$$Cov\left[\sum_{i=1}^{n} a_i X_i, \sum_{j=1}^{m} b_j Y_j\right] = \sum_{i=1}^{n}\sum_{j=1}^{m} a_i b_j Cov[X_i, Y_j], \quad \forall a_i, b_j \in R$$

Se le VA $(X_1,...,X_n)$ sono incorrelate $Var\left[\sum_{i=1}^{n} X_i\right] = \sum_{i=1}^{n} Var[X_i]$

FVC: $Var[X] = E[Var[X \mid Y]] + Var[E[X \mid Y]]$

TLC: Sia $S_n = \sum_{i=1}^{n} X_i$, con X_i IID con media η_X e varianza $\sigma_X{}^2$

Allora $F_{S_n}(s) = P\{S_n \le s\} \approx 1 - Q\left(\frac{s - n\eta_X}{\sigma_X \sqrt{n}}\right)$

Se X_i variabili discrete a reticolo $P\{S_n = ka\} = a g_S(ka)$, con

$$g_S(x) = \frac{1}{\sqrt{2\pi}\sigma_S} e^{-\frac{(x-\eta_S)^2}{2\sigma_S{}^2}} .$$

Approssimazioni binomiali

$$P\{S_n = k\} \approx \frac{e^{-\frac{(k-np)^2}{2np(1-p)}}}{\sqrt{2\pi np(1-p)}} \quad \text{se} \quad np(1-p) >> 1$$

$$P\{S_n = k\} \approx \frac{(np)^k}{k!} e^{-np} \quad \text{se} \quad np(1-p) \le 1$$

Somma stocastica: $Z = \sum_{i=1}^{N} X_i$ con $E[X_i] = \eta_X$ $E[N] = \eta_N$, con

N indipendente dagli X_i. $\qquad Var[Z] = \sigma_X{}^2 \eta_N + \eta_X{}^2 \eta_N{}^2$

$$f_{XY}(x,y) = \frac{e^{-\frac{1}{2(1-\rho^2)}\left[\frac{(x-\eta_X)^2}{\sigma_X^2} - 2\rho\frac{(x-\eta_X)(y-\eta_Y)}{\sigma_X\sigma_Y} + \frac{(y-\eta_Y)^2}{\sigma_Y^2}\right]}}{2\pi\sigma_X\sigma_Y\sqrt{1-\rho^2}}$$

Disug. di Schwartz: $E[XY]^2 \le E[X^2]E[Y^2]$

Disug. Chebychev $P\{(X - \eta_X) \ge \varepsilon\} \le \frac{\sigma_X{}^2}{\varepsilon^2}$

$$P\{(X - \eta_X) < \varepsilon\} > 1 - \frac{\sigma_X{}^2}{\varepsilon^2}$$

Teorema Momenti: $E[X^n] = \phi_X{}^{(n)}(0)$

Dove $\phi_X(s) = E[e^{sX}] = \int_{-\infty}^{+\infty} e^{sx} f_X(x)dx$

Uso notevole della CDF Congiunta:

$$P\{X > x_1, Y > y_1\} = 1 - \left[F_{XY}(\infty, y_1) + F_{XY}(x_1, \infty) - F_{XY}(x_1, y_1)\right]$$